

# Public Investment and Debt Sustainability in Low-Income Countries

## *The DIG Model*

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# Outline

## ① The Debt, Public Investment, and Growth (DIG) Model

- Agents' decisions
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## ② Policy Experiments

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- The Medium-run dynamics
- Risks

## ③ Country Applications and Extensions

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- Investing in electricity generation in Ethiopia and Lesotho
- Improving revenue collection in Cambodia
- Investing in schools vs. roads
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- Explaining growth projections in Togo

## Motivation: Is public investment always good?

- The government intends to build this:



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- But got that instead:



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## Reference

- A complete description of the **DIG** model:
  - Buffie, E., A. Berg , C. Pattillo, R. Portillo and L.F. Zanna (2012), “Public Investment, Growth, and Debt Sustainability: Putting Together the Pieces,” IMF WP 12/144 [[here](#)].

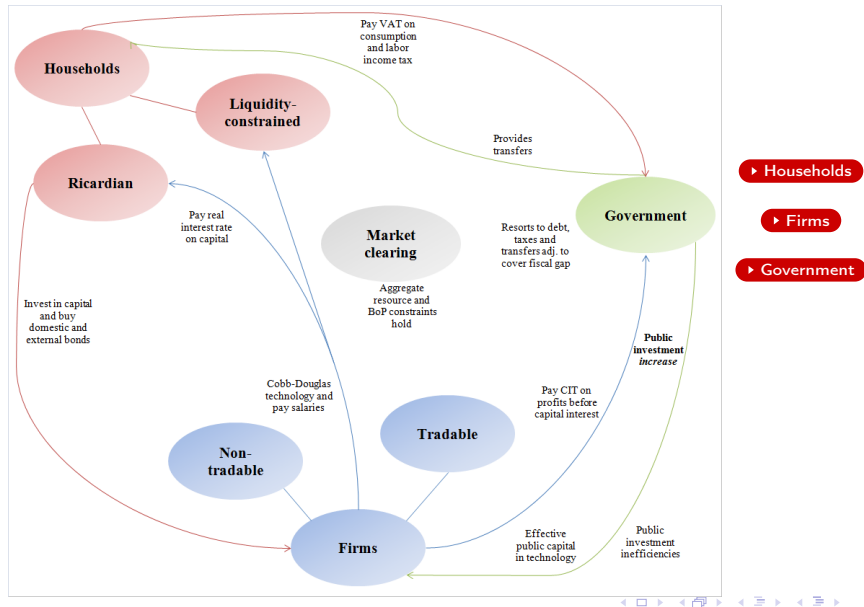
# The model

*"All models are wrong, but some are useful" (George P.E. Box)*

- A real micro-founded dynamic equilibrium model of a small open economy where agents have perfect foresight.
- Exogenous growth ( $g$ ). Variables are detrended by  $(1 + g)^t$ .
- A composite good produced abroad is the numeraire ( $P_t^* = 1$ ).
- 2 sectors: traded ( $q_{x,t}$ ) and non-traded ( $q_{n,t}$ ).
- 3 consumption goods: a traded-domestically-produced good ( $c_x$ ), a traded-foreign good ( $c_m$ ) and a non-traded good ( $c_n$ ).
- 2 types of households: savers ( $s$ ) and non-savers ( $h$ ) where  $a = \frac{L^h}{L^s}$ .



# The economy in a snapshot



# Firms

- There is a continuum of perfectly competitive firms, which are profit maximizers, producing tradable and nontradable goods.
- They have a Cobb-Douglas production function, enhanced by total factor productivity, with productive public and private capital and labor as inputs.
- Labor is mobile across sectors and returns on private capital are sector-specific (off steady state).

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## Firms

- The traded sector ( $x$ ) is profit-maximizing, operates under perfect competition and flexible prices, and firms combine labor ( $L_{x,t}$ ), private capital ( $k_{x,t}$ ), and public capital ( $z_t^e$ ).

$$q_{x,t} = A_{x,t} (z_{t-1}^e)^\psi (k_{x,t-1})^{\alpha_x} (L_{x,t})^{1-\alpha_x}$$

- Similar technology in the non-traded sector ( $n$ ).

$$q_{n,t} = A_{n,t} (z_{t-1}^e)^\psi (k_{n,t-1})^{\alpha_n} (L_{n,t})^{1-\alpha_n}$$

- Firms' total factor productivity  $A_j$  could include a learning-by-doing externality capturing the Dutch disease effect.
  - When the externality is greater in the traded sector than in the non-traded sector—i.e. a decline in the traded sector output imposes an economic cost through a sectoral loss in TFP.

# Firms

## Factor Demands:

- Traded Sector

$$P_{x,t}(1 - \alpha_x) \frac{q_{x,t}}{L_{x,t}} = w_t \quad \text{and} \quad P_{x,t} \alpha_x \frac{q_{x,t}}{k_{x,t-1}} = r_{x,t}$$

- Non-traded sector

$$P_{n,t}(1 - \alpha_n) \frac{q_{n,t}}{L_{n,t}} = w_t \quad \text{and} \quad P_{n,t} \alpha_n \frac{q_{n,t}}{k_{n,t-1}} = r_{n,t}$$

$P_x$  and  $P_n \equiv$  Relative price of the “x” and “n” goods,  $w \equiv$  Wage

$r_x$  and  $r_n \equiv$  Rental earned by capital in sector  $x$  and  $n$

## Producing private capital and infrastructure

- Labor is mobile intersectorally while capital is sector-specific capital, but  $r_x \neq r_n$  *only* on transition.
- Supply: Private capital ( $k$ ) and infrastructure ( $z$ ) are built by combining one imported machine with  $a_j$  ( $j = k, z$ ) units of a non-traded input.
- Supply (relative) prices of private capital and infrastructure are

$$P_{k,t} = P_{mm,t} + a_k P_{n,t} \quad \text{and} \quad P_{z,t} = P_{mm,t} + a_z P_{n,t}$$

$P_{mm} \equiv$  price of imported machinery (normalized to 1).

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# Households

- The economy is populated by a continuum of infinitely-lived households of two types:
  - Ricardian households (or Savers) maximize their expected lifetime utility over consumption and leisure. These have access to domestic and external borrowing, and can save by accumulating capital, which they rent to firms in both tradable and nontradable sectors.
  - Liquidity-constrained households (or Non-savers) can only consume as much as their income from wages, remittances, and transfers each period allows.
- Households' consumption is composed of tradable goods produced domestically and abroad, and domestic nontradable goods.
- Both households pay consumption taxes (VAT-like). Ricardian households pay additional fees for using public infrastructure goods and services.

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## Households: Savers

Savers ( $s$ ) are inter-temporal maximizing

$$\text{Max} \sum_{t=0}^{\infty} \beta^t \frac{(c_t^s)^{1-\frac{1}{\tau}}}{1-\frac{1}{\tau}} \quad \text{where} \quad c_t^s = \left[ \sum_{i=x,m,n} \rho_i^{\frac{1}{\varepsilon}} (c_{i,t}^s)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

- Access to domestic bonds  $b_t^s (r_t)$  and foreign bonds  $b_t^{s*} (r_t^*)$
- Accumulate capital  $i_{j,t}^s$  for  $j = x, n$
- Pay VAT  $h_t$  and user fees  $\mu z_{t-1}^e$
- Receive remittances  $\mathcal{R}_t^s$ , net transfers  $\mathcal{T}_t^s$ , labor income  $w_t L_t^s$ , capital income  $\sum_{j=x,n} r_{j,t-1} k_{j,t-1}^s$ , and profits  $\Phi_t^s$

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## Households: Savers

Budget Constraint

$$P_t b_t^s - b_t^{s*} + P_{k,t} \sum_{j=x,n} (i_{j,t}^s + AC_{j,t}^s) + P_t c_t^s (1 + h_t) + \mu z_{t-1}^e + \mathcal{P}_t^s = w_t L_t^s$$

$$+ \sum_{j=x,n} r_{j,t-1} k_{j,t-1}^s + \mathcal{R}_t^s + \mathcal{T}_t^s + \Phi_t^s + \frac{1+r_{t-1}}{1+g} P_t b_{t-1}^s - \frac{1+r_{t-1}^*}{1+g} b_{t-1}^{s*}$$

with  $P_t = \left[ \sum_{i=x,m,n} \rho_i P_{i,t}^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}$  and adjustment costs (associated with changing the capital stock and foreign liabilities)

$$AC_{j,t}^s \equiv \frac{v}{2} \left( \frac{i_{j,t}^s}{k_{j,t-1}^s} - \delta - g \right)^2 k_{j,t-1}^s \quad \text{for } j = x, n$$

$$\mathcal{P}_t^s \equiv \frac{\eta}{2} (b_t^{s*} - \bar{b}^{s*})^2$$



## Households: Savers

- Capital accumulation (for  $j = x, n$ )

$$(1 + g)k_{j,t}^s = (1 - \delta)k_{j,t-1}^s + i_{j,t}^s$$

- Some optimal conditions (for  $j = x, n$ )

$$\underbrace{\frac{c_{t+1}^s}{c_t^s} = \left[ \beta \left( \frac{1 + r_t}{1 + g} \right) \frac{1 + h_t}{1 + h_{t+1}} \right]^\tau}_{\text{Consumption Euler equation}}$$

Consumption Euler equation

$$\underbrace{\frac{r_{j,t+1}}{Q_{j,t}} + (1 - \delta) \frac{Q_{j,t+1}}{Q_{j,t}} - \frac{P_{k,t+1}}{Q_{j,t}} \frac{\partial AC_{j,t+1}^s}{\partial k_{j,t}^s}}_{\text{Sectoral returns on capital equal real interest rate on domestic bonds}} = (1 + r_t) \frac{P_{t+1}}{P_t}$$

Sectoral returns on capital equal real interest rate on domestic bonds

where  $Q_{j,t} = P_{k,t} \left[ 1 + v \left( \frac{i_{j,t+1}^s}{k_{j,t}^s} - \delta - g \right) \right]$ .

## Households: Savers

- Some optimal conditions

$$(1 + r_t) \frac{P_{t+1}}{P_t} = \frac{1 + r_t^*}{1 - \eta(b_t^{s*} - \bar{b}^{s*})}$$

$\eta$  measures the degree of openness of the private CA

- A constant premium  $u$  over the interest rate that the government pays on external commercial debt  $r_{dc}$

$$r_t^* = r_{dc,t} + u$$

- $u$  to match the low capital in LICs. At the steady state

$$\bar{r}_{dc} + u = \bar{r}^* = \bar{r} \implies \text{High } \bar{r} \implies \text{High } MPk_j \implies \text{Low } k_j$$

## Households: Non-savers

Non-savers ( $h$ ) live check-to-check and solve

$$\text{Max} \sum_{t=0}^{\infty} \beta^t \frac{(c_t^h)^{1-\frac{1}{\tau}}}{1-\frac{1}{\tau}} \quad \text{where} \quad c_t^h = \left[ \sum_{i=x,m,n} \rho_i^{\frac{1}{\varepsilon}} (c_{i,t}^h)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

subject to

$$P_t c_t^h (1 + h_t) = w_t L_t^h + R_t^h + \mathcal{T}_t^h$$

- No access to assets.

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# Government

- After scaling up public investment, the government is allowed to have a fiscal deficit that can be financed through a combination of debt instruments (domestic, concessional, and external commercial) and fiscal adjustment (taxes or transfers).
- *Government revenues*: VAT on households' consumption and fees collected from savers' usage of infrastructure services.
- *Government expenditures*: Transfers, debt service, and infrastructure investment.
- Public investment is inefficient and can include costs associated with absorptive capacity constraints.

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# Government

- Invests  $(i_{z,t})$  in public capital with some inefficiencies  $(s, \bar{s})$  (Hulten, 1996, and Pritchett, 2000....)

$$(1+g)z_t^e = (1-\delta)z_{t-1}^e + \underbrace{\bar{s}}_{\text{historical investment}} + s \underbrace{(i_{z,t} - \bar{s})}_{\text{investment scaling-up}}$$

with  $s, \bar{s} \in [0, 1]$  and  $\bar{s} = (\delta + g)\bar{z}$ .

- Public investment outlays  $P_{z,t} \mathbb{I}_{z,t}$  include costs overruns associated with absorptive capacity constraints

$$\mathbb{I}_{z,t} = \mathcal{H}_t(i_{z,t} - \bar{s}) + \bar{s}$$

with  $\mathcal{H}_t = \left(1 + \frac{i_{z,t}}{z_{t-1}} - \delta - g\right)^\phi$ , where  $\phi \geq 0$

## Government

- Charges taxes on consumption ( $h_t$ ), provides lump-sum transfers ( $\mathcal{T}_t$ ), collects user fees ( $\mu z_{t-1}^e$  with  $\mu = fP_{z,o}\delta$ )
- Has access to different financing schemes
  - concessional  $\Delta d_t = d_t - d_{t-1}$  with constant interest rate  $r_d$
  - external commercial  $\Delta d_{c,t} = d_{c,t} - d_{c,t-1}$  with

$$r_{dc,t} = \underbrace{r^f}_{\text{free-risk world rate}} + \underbrace{v_g e^{\eta_g \left( \frac{d_t + d_{c,t}}{y_t} - \frac{\bar{d} + \bar{d}_c}{\bar{y}} \right)}}_{\text{risk premium}},$$

- domestic  $\Delta b_t = b_t - b_{t-1}$  with

$$r_t = \frac{1+g}{\beta} \left( \frac{1+h_{t+1}}{1+h_t} \right) \left( \frac{c_{t+1}^s}{c_t^s} \right)^{1/\tau} - 1$$

# The government budget constraint (GBC)

Simplifying  $g = 0$  and  $\phi = 0$

$$\begin{aligned}
 P_t \Delta b_t + \Delta d_{c,t} + \Delta d_t &= \underbrace{r_{t-1} P_t b_{t-1} + r_d d_{t-1} + r_{dc,t-1} d_{c,t-1}}_{\text{Interest Payments}} \\
 &+ \underbrace{P_{z,t} i_{z,t}}_{\text{Investment}} + \underbrace{\mathcal{I}_t}_{\text{Transfers}} - \underbrace{h_t P_t c_t}_{\text{Taxes}} \\
 &- \underbrace{\mu z_{t-1}^e}_{\text{User fees}} - \underbrace{\mathcal{N}_t}_{\text{NR revenues}} - \underbrace{\mathcal{G}_t}_{\text{Grants}}
 \end{aligned}$$

$\Delta d_t = d_t - d_{t-1}$ : concessional loans

$\Delta d_{c,t} = d_{c,t} - d_{c,t-1}$ : external commercial loans

$\Delta b_t = b_t - b_{t-1}$ : domestic loans

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## The government budget constraint (GBC)

- Given public investment and concessional borrowing, the fiscal gap *before* policy adjustment

$$\begin{aligned} \text{Gap}_t = & r_d d_{t-1} - \Delta d_t + r_{dc,t-1} d_{c,t-1} + r_{t-1} P_t b_{t-1} \\ & + P_{z,t} i_{z,t} + \mathcal{I}_o - h_o P_t c_t - \mathcal{G}_t - \mathcal{N}_t - \mu z_{t-1}^e \end{aligned}$$

- Then GBC

$$\text{Gap}_t = P_t \Delta b_t + \Delta d_{c,t} + (h_t - h_o) P_t c_t - (\mathcal{I}_t - \mathcal{I}_o)$$

- Debt sustainability requires  $h_t$  and  $\mathcal{I}_t$  eventually adjust to cover gap (i.e.,  $P_t \Delta b_t + \Delta d_{c,t} = 0$  for  $t \gg 0$ )



# Fiscal adjustment and reaction functions

- Over time, the fiscal burden falls on  $h_t$  and  $\mathcal{T}_t$  with a policy parameter  $\lambda$  that splits the adjustment
  - If  $\lambda = 0$  then the burden falls exclusively on taxes  $h_t$
  - If  $\lambda = 1$  then the burden falls solely on transfers  $\mathcal{T}_t$
- For this course, assume all the burden falls on  $h_t$  (i.e.,  $\lambda = 0$ )

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# Fiscal adjustment and reaction functions

- Concessional borrowing  $\Delta d_t$  is exogenous (also  $i_{z,t}$ ,  $\mathcal{G}_t$ , and  $\mathcal{N}_t$ )
- Types of financing schemes that we will study
  - Unconstrained tax adjustment:  $h_t$  adjusts to satisfy GBC (gap) and not need to borrow  $\Delta d_{c,t} = 0$  and  $\Delta b_t = 0$
  - Constrained tax adjustment supplemented with external commercial borrowing. Cap on  $h_t$  and  $\Delta b_t = 0$
  - Constrained tax adjustment supplemented with domestic borrowing. Cap on  $h_t$  and  $\Delta d_{c,t} = 0$

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# Unconstrained tax adjustment

- Reminders
  - Concessional borrowing  $\Delta d_t$  is exogenous (also  $i_{z,t}$ ,  $\mathcal{G}_t$ , and  $\mathcal{N}_t$ )
  - The burden falls on taxes and keep  $\mathcal{T}_t = \mathcal{T}_o$
- No need to borrow domestically or commercially  $\Delta d_{c,t} = 0$  and  $\Delta b_t = 0$
- In the long run and short/medium run  $h_t$  adjusts to satisfy GBC

$$\text{Gap}_t = (h_t - h_o)P_t c_t$$

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# Constrained tax adjustment supplemented with external commercial borrowing

- $\mathcal{I}_t = \mathcal{I}_o$ , and  $b_t = b_o$ , so

$$\text{Gap}_t = \Delta d_{c,t} + (h_t - h_o)P_t c_t$$

- In the long run  $h_t$  adjusts to cover the gap (i.e.,  $\Delta d_{c,t} = 0$  for  $t \gg 0$ ).
- How is  $h_t$  determined in the short/medium run? By a fiscal reaction function.

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# Constrained tax adjustment supplemented with external commercial borrowing

- In the short/medium run

$$h_t = \text{Min} \left\{ h_t^{\text{rule}}, h^u \right\}$$

$$h_t^{\text{rule}} = h_{t-1} + \lambda_1 (h_t^{\text{target}} - h_{t-1}) + \lambda_2 \frac{(d_{c,t-1} - d_c^{\text{target}})}{y_t}$$

- $\lambda_1, \lambda_2 > 0$ ,
- $h^u$  is a cap (ceiling)
- $h_t^{\text{target}}$  is the VAT that would be required to satisfy the GBC when  $\Delta d_{c,t} = 0$   
 (i.e.,  $h_t^{\text{target}} = h_o + \frac{\text{Cap}_t}{P_t C_t}$ )

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## Constrained tax adjustment supplemented with external commercial borrowing

- If  $h_t$  adjustment is delayed (lower  $\lambda_1$ ) or the cap is tight (low  $h^u$ ) then the government has to borrow  $\Delta d_{c,t}$

$$\mathfrak{G}ap_t = \Delta d_{c,t} + (h_t - h_o)P_t c_t$$

- Why can debt be explosive?
  - If the government moves too slowly ( $\lambda_1$  is too low) or the cap constrains adjustment too much (very low  $h^u$ ), interest rate payments will rise faster than revenue net of transfers (increasing  $\mathfrak{G}ap_t$ ) causing debt to grow explosively.

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## Constrained tax adjustment supplemented with domestic borrowing

- $\mathcal{I}_t = \mathcal{I}_0$ , and  $d_{c,t} = d_{c0}$ , so

$$\text{Gap}_t = P_t \Delta b_t + (h_t - h_0) P_t c_t$$

- In the long run  $h_t$  adjusts to cover the gap (i.e.,  $P_t \Delta b_t = 0$  for  $t \gg 0$ ).
- In the short/medium run

$$h_t = \text{Min} \left\{ h_t^{\text{rule}}, h^u \right\}$$

$$h_t^{\text{rule}} = h_{t-1} + \lambda_1 (h_t^{\text{target}} - h_{t-1}) + \lambda_2 \frac{(b_{t-1} - b^{\text{target}})}{y_t}$$

- $\lambda_1, \lambda_2 > 0$ ,  $h^u$  is a cap, and  $h_t^{\text{target}}$  is the VAT that would be required to satisfy the GBC when  $\Delta b_t = 0$ .

## What if the burden also falls on transfers?

- Similar to tax specification (remember  $\lambda$ ).
- In the short/medium run

$$\mathcal{T}_t = \text{Max} \left\{ \mathcal{T}_t^{\text{rule}}, \mathcal{T}^l \right\}$$

$$\mathcal{T}_t^{\text{rule}} = \mathcal{T}_{t-1} + \lambda_3(\mathcal{T}_t^{\text{target}} - \mathcal{T}_{t-1}) - \lambda_4(x_{t-1} - x^{\text{target}})$$

- $\lambda_1, \lambda_2 > 0$ ,
- $\mathcal{T}^u$  is a floor
- $\mathcal{T}_t^{\text{target}}$  is the required transfers to help cover the gap
- $x_t = b_t$  or  $d_{c,t}$ .

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## Market clearing

- Labor

$$L_{x,t} + L_{n,t} = L_t^s + L_t^h$$

- Non-traded goods

$$q_{n,t} = \rho_n \left( \frac{P_{n,t}}{P_t} \right)^{-\varepsilon} c_t + a_k \sum_{j=x,n} (i_{j,t} + AC_{j,t}) + a_z l_{z,t}$$

- Resource constraint

$$\begin{aligned} \Delta d_t + \Delta d_{c,t} + \Delta b_t^* &= \frac{r_d - g}{1 + g} d_{t-1} + \frac{r_{dc,t-1} - g}{1 + g} d_{c,t-1} + \frac{r_{t-1}^* - g}{1 + g} b_{t-1}^* \\ &+ \mathcal{P}_t + P_{z,t} \mathbb{I}_{z,t} + P_{k,t} \sum_{j=x,n} (i_{j,t} + AC_{j,t}) \\ &+ P_t c_t - P_{n,t} q_{n,t} - P_{x,t} q_{x,t} - \mathcal{R}_t - \mathcal{Y}_t - \mathcal{N}_t \end{aligned}$$

## Dynamic equations

$$\frac{c_{t+1}^s}{c_t^s} = \left[ \beta \left( \frac{1+r_t}{1+g} \right) \frac{1+h_t}{1+h_{t+1}} \right]^\tau$$

$$(1+g)k_{j,t}^s = (1-\delta)k_{j,t-1}^s + i_{j,t}^s$$

$$q_{j,t} = A_{j,t} (z_{t-1}^e)^\psi (k_{j,t-1})^{\alpha_j} (L_{x,t})^{1-\alpha_j}$$

$$A_{j,t} = a_x \left( \frac{q_{j,t-1}^l}{\bar{q}_j^l} \right)^{\sigma_j} \left( k_{j,t-1}^l \right)^{\xi_j}$$

$$(1+r_t) \frac{P_{t+1}}{P_t} = \frac{1+r_t^*}{1-\eta(b_t^{s*} - b^{s*})}$$

$$(1+g)z_t^e = (1-\delta)z_{t-1}^e + \bar{s}\bar{l}_z + s(i_{z,t} - \bar{l}_z)$$

$$P_{j,t}(1-\alpha_j) \frac{q_{j,t}}{L_{j,t}} = w_t$$

$$P_{j,t} \alpha_j \frac{q_{j,t}}{k_{x,t-1}} = r_{j,t}$$

$$h_t^{\text{rule}} = h_{t-1} + \lambda_1 (h_t^{\text{target}} - h_{t-1}) + \lambda_2 \frac{(d_{c,t-1} - d_c^{\text{target}})}{y_t}$$

$$h_t^{\text{target}} = h_o + \frac{\mathcal{G}_{\text{cap}_t}}{P_t c_t}$$

$$\mathcal{I}_t^{\text{rule}} = \mathcal{I}_{t-1} + \lambda_3 (\mathcal{I}_t^{\text{target}} - \mathcal{I}_{t-1}) - \lambda_4 (x_{t-1} - x^{\text{target}})$$

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## Calibration to an "average" LIC

- The trend growth rate ( $g$ ) of 1.5% equals the 1990-2008 per capita growth rate for SSA.
- Intertemporal elasticity of substitution  $\tau = 0.34$  (Ogaki et al., 2000).
- Distribution parameters  $\rho_n = 0.43$  and  $\rho_m = 0.37$  to match shares of non-tradables in GDP (50%) and imports in GDP (45%). LICs averages 1998-2008.
- Capital's shares in value added  $\alpha_x = 0.40$  and  $\alpha_n = 0.55$  (GTAP social accounting matrices).
- No externalities  $\sigma_j = \xi_j = 0$  for  $j = x, n$ .
- No risk premium on higher external public debt ( $\eta_g = 0$ ).

- Closed (private) capital account ( $\eta = 1$ ).
- No absorptive capacity constraints ( $\phi = 0$ ).
- Public external debt to GDP is 50% of GDP and domestic debt is 20% of GDP (IMF, 2009).
- Real interest rates on concessional debt,  $r_{do} = 0$ , external commercial debt,  $r_{dc,o} = 0.065$ , and domestic debt,  $r_o = 0.1$ , (IMF-WB, DSAs 2009)
- Grants ( $\mathcal{G}_o$ ) equal 5% of GDP (Barkbu et al., 2008).
- Consumption VAT  $h_o = 0.15$  and transfers ( $\mathcal{T}_o$ ) clear the GBC.

- Initial public investment to GDP  $\frac{P_{z,o}i_{z,o}}{y_o} = 6\%$
- Efficiency  $s = 0.6$  (Pritchett, 2000)
- Depreciation is about 5%
- Return on public capital  $R = MP_z(s, \psi, \dots) - \delta = 0.25$  (Dalgaard and Hansen, 2005, Foster and Briceño-Garmendia, 2010)
- User fees recoup 50% of recurrent costs  $f = 0.5$  (Briceño-Garmendia et al., 2008).  $\mu z_{t-1}^e$  with  $\mu = fP_{z,o}\delta$
- Policy parameters  $(\lambda, \lambda_1, \lambda_2, h^u \dots)$  values depend on the experiments
- Other parameters see Buffie et al. (2012)

A front-loaded surge which in the long run reaches 3% of GDP and only 50% of it is covered by concessional loans and grants.

### Public Investment, Concessional Borrowing, and Grants

| Year                                | 1 | 2  | 3  | 4  | 5   | 6   | 7  | 8   | 9  | 10.. | 29.. | 32... |
|-------------------------------------|---|----|----|----|-----|-----|----|-----|----|------|------|-------|
| $\frac{P_{z,o}\Delta i_{z,t}}{y_o}$ | 0 | 5  | 7  | 7  | 6.6 | 5.8 | 5  | 4.4 | 4  | 3    | 3    | 3     |
| $\frac{\Delta d_t}{y_o}$            | 0 | 4  | 5  | 4  | 3   | 2   | 1  | .8  | .5 | -1   | 0    | 0     |
| $\frac{\Delta \mathcal{G}_t}{y_o}$  | 0 | .4 | .4 | .4 | .4  | .4  | .4 | .4  | .4 | .2   | .2   | 0     |

Note: figures are in percent of initial GDP ( $y_o = 100$ ).

# Solving the Model

- The model takes the following exogenous paths as given
  - Public investment scaling-up
  - Concessional loans
  - Grants
  - Remittances
  - Other shocks
- The inputs—exogenous paths and parameters—are specified in Excel
- The model runs in Matlab, calling Dynare to find a perfect foresight, non-linear solution

## The long-run outcome

Public Investment surges may be self-financing in the long run (dynamic scoring), depending on structural conditions

Long-run Effects of  $P_{z,o}\Delta i_z$  of 3% of  $GDP_o$

|                 | Base Case                | Optimistic             | Troublesome              |
|-----------------|--------------------------|------------------------|--------------------------|
|                 | $R = .25$                | $R = .35$              | $R = .1$                 |
|                 | $s = .6$                 | $s = 1$                | $s = .2$                 |
|                 | $\mu = .05$ ( $f = .5$ ) | $\mu = .1$ ( $f = 1$ ) | $\mu = .02$ ( $f = .2$ ) |
| Taxes (%)       | 16.29                    | 12.81                  | 18.73                    |
| CIC             | 1.51                     | 1.91                   | 0.85                     |
| Public Capital  | 50.00                    | 83.33                  | 16.67                    |
| Private Capital | 13.23                    | 27.98                  | 2.49                     |
| Real GDP        | 13.45                    | 28.50                  | 2.48                     |
| Real Wages      | 13.53                    | 28.76                  | -1.24                    |
| Consumption     | 9.29                     | 23.73                  | -3.33                    |



# Experiments: The medium-run dynamics

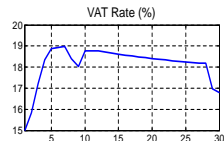
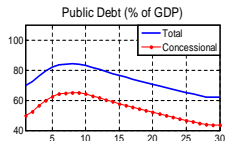
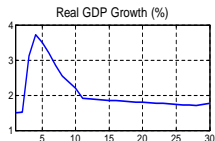
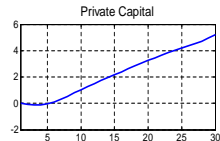
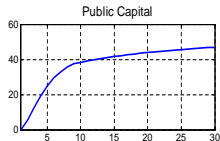
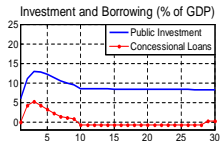
- 3 types of experiments (fiscal burden falls on taxes)
  - Unconstrained tax adjustment
  - Constrained tax adjustment supplemented with external commercial borrowing
  - Constrained tax adjustment supplemented with domestic borrowing

# Unconstrained tax adjustment

- Even when the long run looks good, transition problems can be formidable when concessional financing does not cover the full cost of the investment program (these are high-yielding projects!)
- Covering the resulting gap with tax increases (or spending cuts) requires sharp macroeconomic adjustments: the required fiscal adjustment crowds out private investment and consumption and delays the benefits of the public investment surge

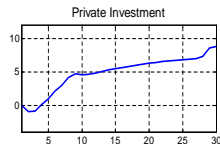
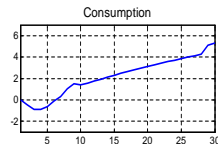
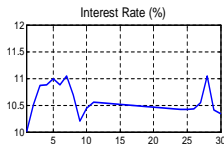
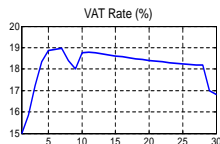
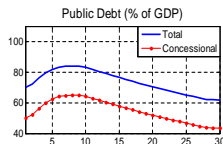
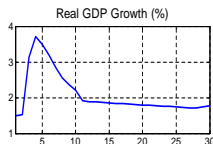
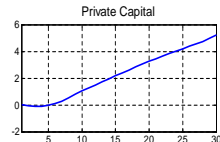
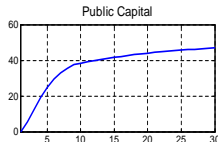
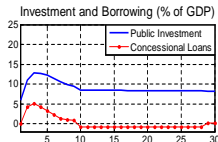
# Unconstrained tax adjustment

Challenging transition even if these are good projects



# Unconstrained tax adjustment

Challenging transition even if these are good projects



# Unconstrained tax adjustment

- It is possible to recreate other scenarios
- Optimistic and troublesome scenarios by changing
  - Return to public capital, efficiency, capacity to collect user fees, absorptive capacity
  - Other parameters: e.g. externalities and Dutch disease

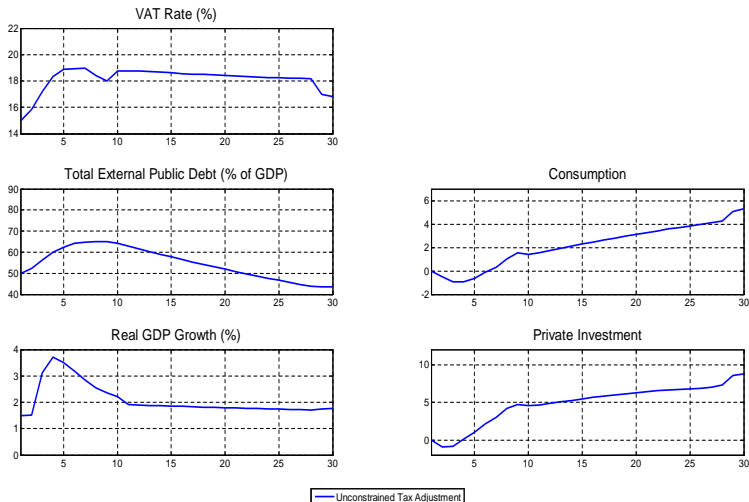
# Constrained tax adjustment

Supplementing with external commercial borrowing can smooth the transition

When concessional financing does not cover the full cost of the investment program, covering the gap with *external commercial borrowing* can smooth the difficult private sector adjustments, while reconciling the scaling ups with constraints on feasible increases in tax rates

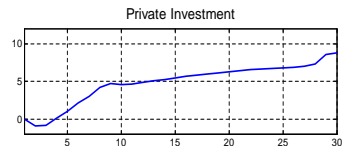
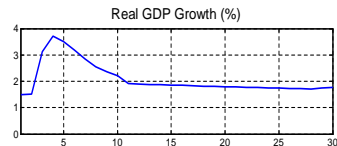
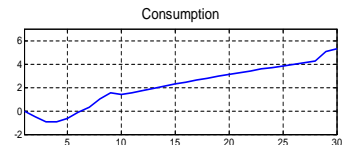
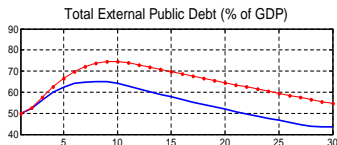
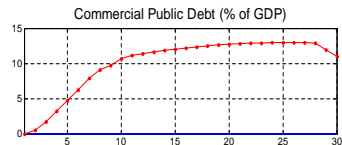
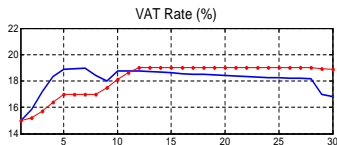
# Constrained tax adjustment

Supplementing with external commercial borrowing can smooth the transition



# Constrained tax adjustment

Supplementing with external commercial borrowing can smooth the transition

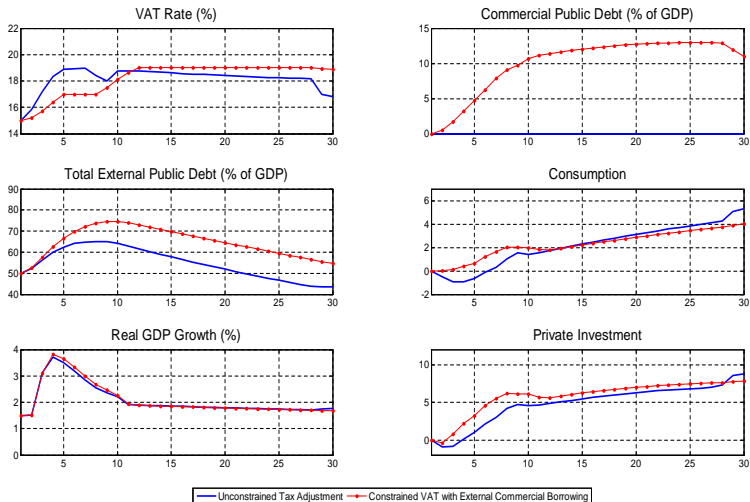


— Unconstrained Tax Adjustment — Constrained VAT with External Commercial Borrowing



# Constrained tax adjustment

Supplementing with external commercial borrowing can smooth the transition



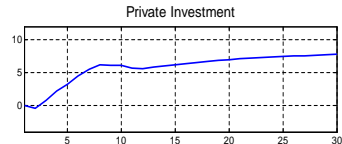
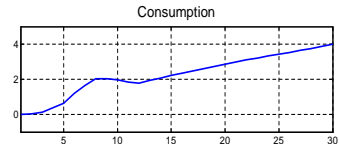
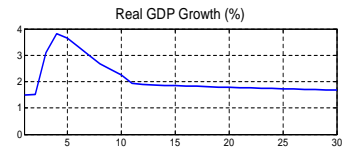
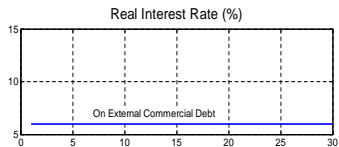
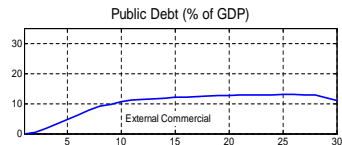
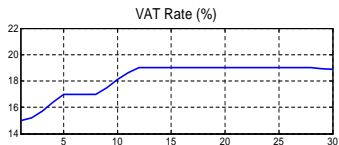
# Constrained tax adjustment

Supplementing with domestic borrowing does *not* help smooth the transition

When concessional financing does not cover the full cost of the investment program, covering the resulting gap with *domestic borrowing* is not helpful: private investment and consumption are still crowded out, since this borrowing does not bring additional resources from abroad and raises domestic interest rates

# Constrained tax adjustment

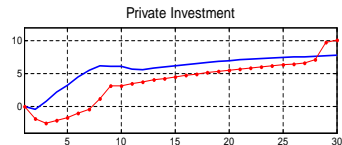
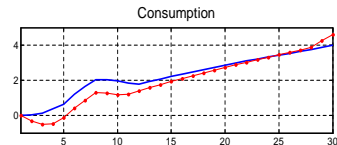
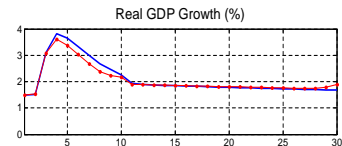
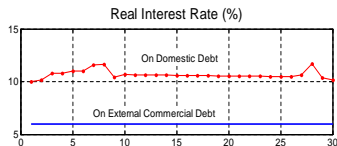
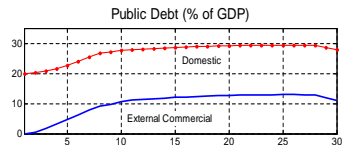
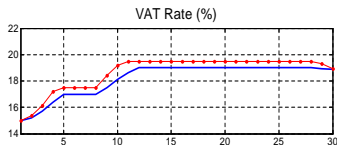
Supplementing with domestic borrowing does *not* help smooth the transition



— External Commercial Borrowing

# Constrained tax adjustment

Supplementing with domestic borrowing does *not* help smooth the transition



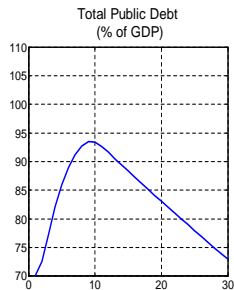
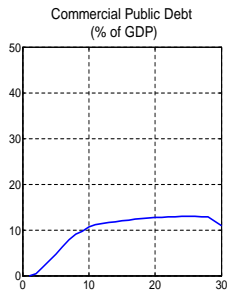
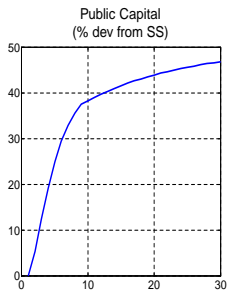
— External Commercial Borrowing — Domestic Borrowing

## Across financing schemes

When concessional financing does not cover the full cost of the investment program, covering the gap with *external commercial borrowing* can smooth the difficult private sector adjustments, while reconciling the scaling ups with constraints on feasible increases in tax rates

# Risks

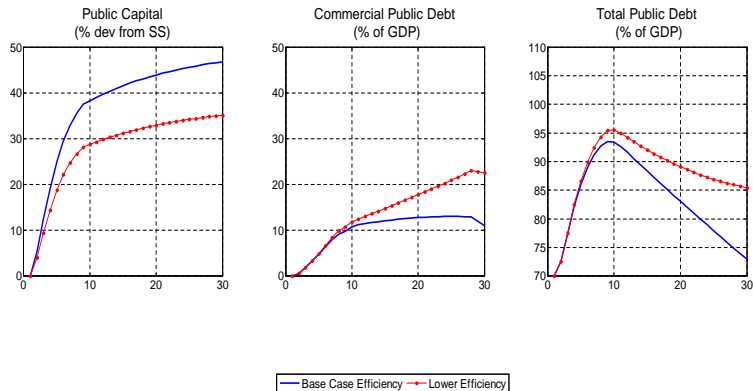
...But it may be risky if efficiency is lower



— Base Case Efficiency

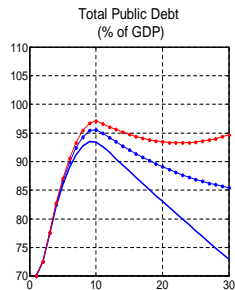
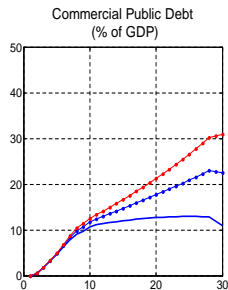
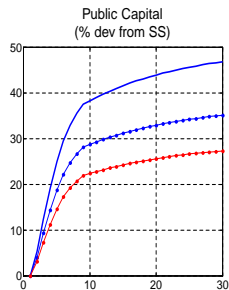
# Risks

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# Risks

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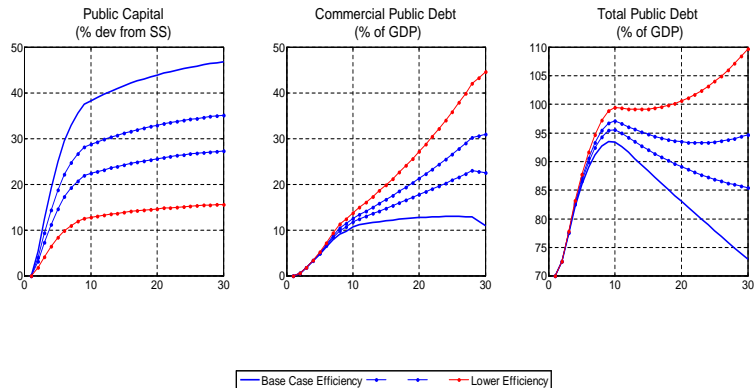


— Base Case Efficiency — Lower Efficiency



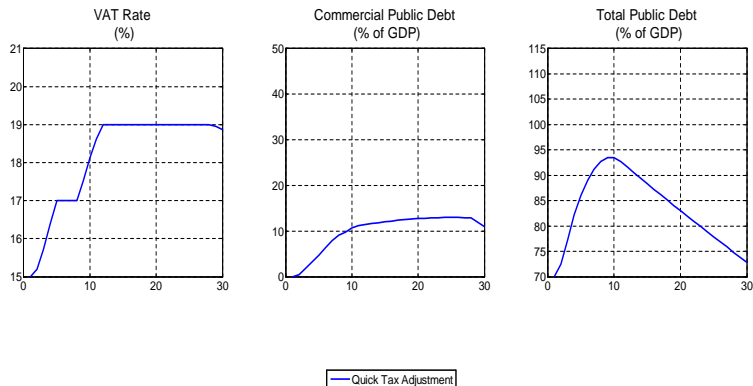
# Risks

...But it may be risky if efficiency is lower



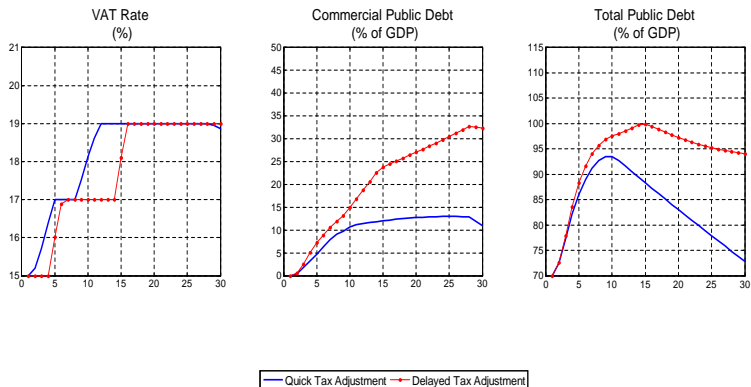
# Risks

...But it may be risky if fiscal adjustment is delayed



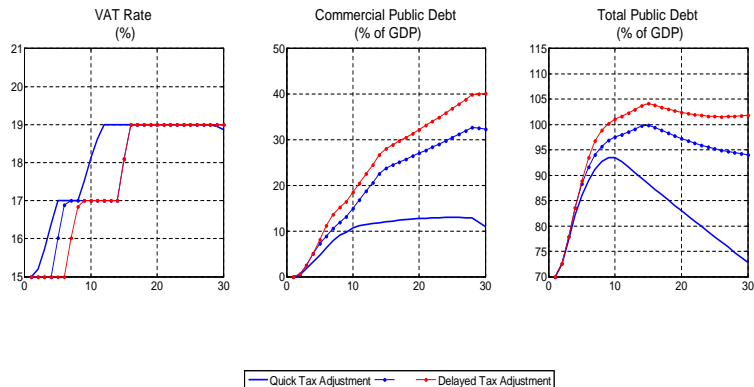
# Risks

...But it may be risky if fiscal adjustment is delayed



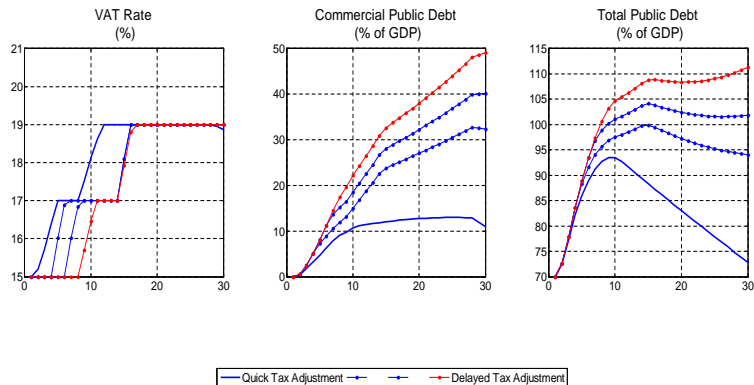
# Risks

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# Risks

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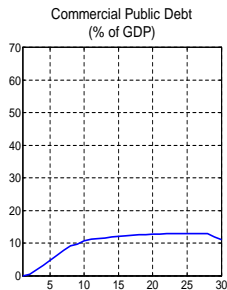
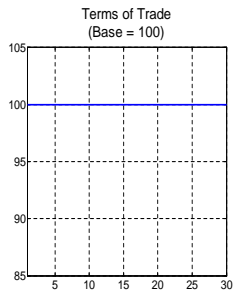


## Other structural conditions

Under external commercial borrowing, similar higher debt sustainability risks are associated with other weak structural conditions (e.g., low return to public capital, low capacity to collect user fees, high absorptive capacity constraints, etc.)

# Risks

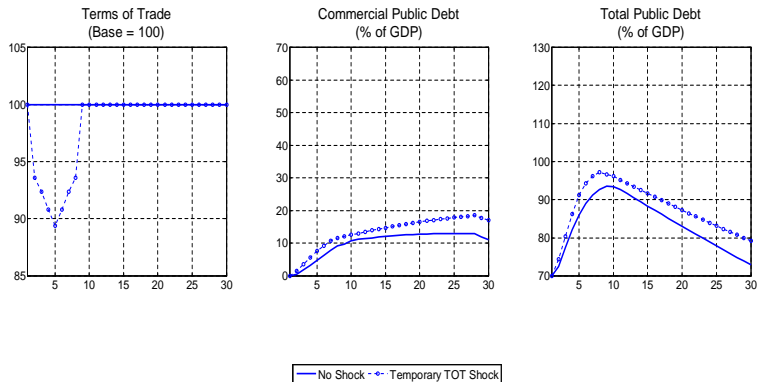
...But it may be risky if there are shocks, especially if persistent



— No Shock

# Risks

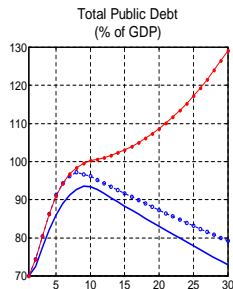
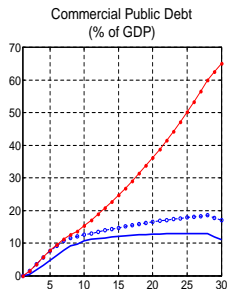
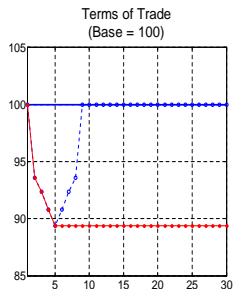
...But it may be risky if there are shocks, especially if persistent





# Risks

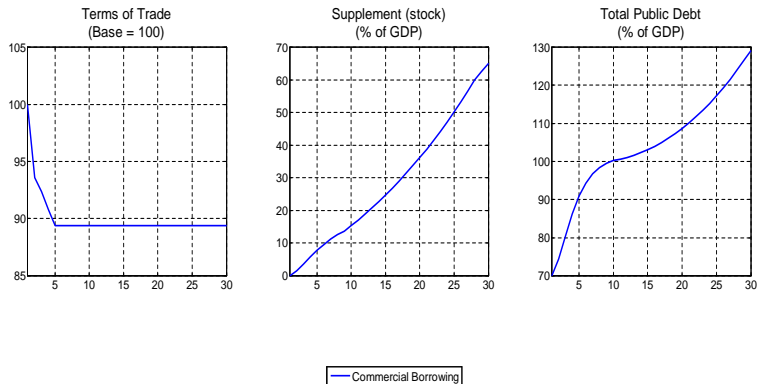
...But it may be risky if there are shocks, especially if persistent



— No Shock —•— Temporary TOT Shock —•— Permanent TOT Shock

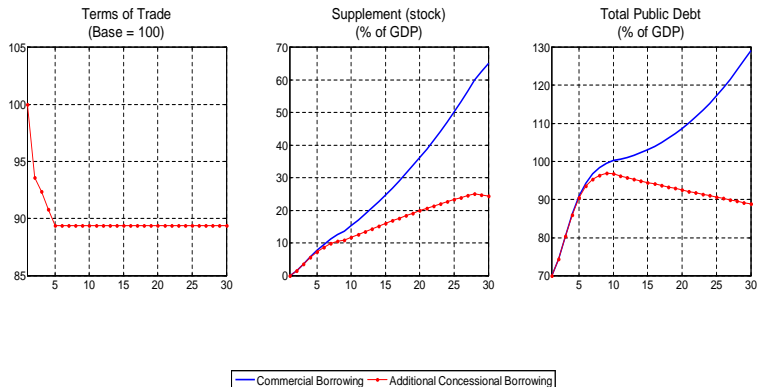
# Risks

With persistent shocks, additional concessional financing, instead of commercial borrowing, may help ensure debt sustainability



# Risks

With persistent shocks, additional concessional financing, instead of commercial borrowing, may help ensure debt sustainability

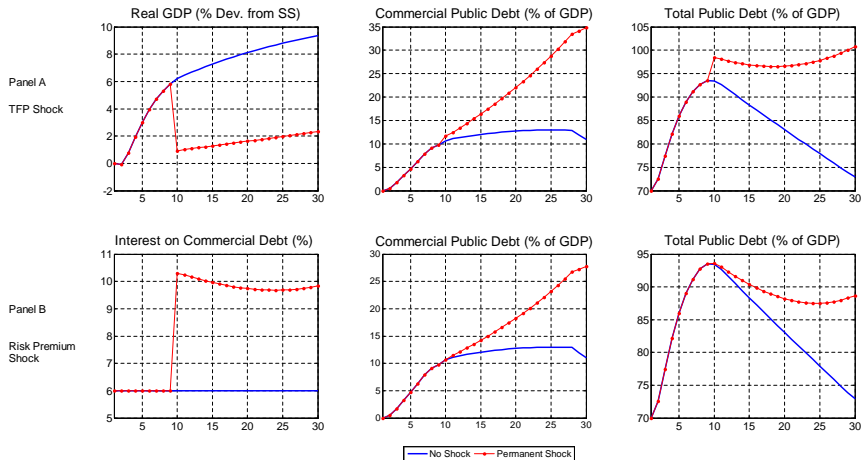


## Other shocks

Under external commercial borrowing, similar higher debt sustainability risks are associated with other persistent negative shocks (e.g., TFP shocks, risk premium shocks, etc.)

# Risks

With other persistent shocks (at different points in time)



## Country applications: Influencing policy work

- Fund staff have applied the model and extensions to countries in the context of Art. IV consultations and program reviews.
  - Completed: Afghanistan, Benin, Burkina Faso, Cape Verde, CEMAC, Cote d'Ivoire, Ethiopia, Ghana, Lesotho, Liberia, Rwanda, Senegal, Togo, and Yemen (and 4 more MIC Arab Countries).
  - Ongoing: Cambodia, Kyrgyz Republic, Maldives, Nigeria, Sri Lanka, Vanuatu.
- This work has complemented the IMF-World Bank DSF by helping country teams and authorities assess the growth, debt, and fiscal implications of ambitious, front-loaded infrastructure investment plans contained in national development plans or PRSPs.

Despite long-run growth benefits of public investment surges, transition macroeconomic challenges and trade-offs accompany the different financing strategies, raising the stakes for policymakers.

- Covering the financing gap with tax increases or spending rationalization requires sharp and unrealistic macroeconomic adjustments, crowding out private investment and consumption in the short to medium term.
  - *Rwanda*: an increase of 5 p.p. of public investment to GDP financed with taxes would lead to a 10 percent decline in consumption relative to its initial level.
- Improving investment efficiency and revenue mobilization are key to reaping growth benefits of scaling-ups while minimizing the associated debt sustainability risks.
  - *Liberia*: a 30% increase in efficiency could deliver a gain of 0.5 p.p. in per-capita growth over the medium term

# The DIGNAR model

- **Debt sustainability, public Investment, Growth and NATural Resources (DIGNAR):** Develop a model to analyze public investment surges, growth, debt sustainability and management of natural resources in resource-rich LICs, using as building blocks the DIG model and the sustainable-investment framework developed in Berg et al. (2013).
  - Melina G., S. Yang, and L.F. Zanna (2016), "Debt Sustainability, Public Investment, and Natural Resources in Developing Countries: The DIGNAR Model," *Economic Modelling*, 52, 630-649.
  - A. Berg, R. Portillo, S. Yang, and L.F. Zanna (2013), "Public Investment in Resource-Abundant Developing Economies," *IMF Economic Review*, Vol. 61(1).
  - Melina G. and Y. Xiong (2013), "Natural Gas, Public Investment, and Debt Sustainability in Mozambique," IMF Working Paper 13/261.
  - Delechat C. et al. (2015), "Harnessing Resource Wealth for Inclusive Growth in Fragile States," IMF Working Paper 15/25.



## Natural resource windfalls can raise the stakes...

- Declining revenues (e.g. several CEMAC countries) raise question of how to convert a finite stock of resources to sustainable development.
  - Analytically, like earlier question of how much to borrow, but risks are in a sense lower.
- Large and sustained revenues (e.g. Angola) present challenge of managing shocks.
  - Avoiding the boom/bust cycle.
- For countries expecting future production, resource revenues can also serve as a collateral for accessing international financial markets.
  - E.g. Mozambique could borrow to accelerate benefits (like the borrowing problem on steroids).

- Now there is a Natural resource sector with resource prices and quantities produced given exogenously such that resource GDP (in units of the domestic consumption basket)

$$y_{o,t} = s_t p_{o,t}^* \tilde{y}_{o,t}$$

- The government collects a royalty tax on the real value of resource production and saves the resource windfall in a sovereign wealth fund that evolves according to

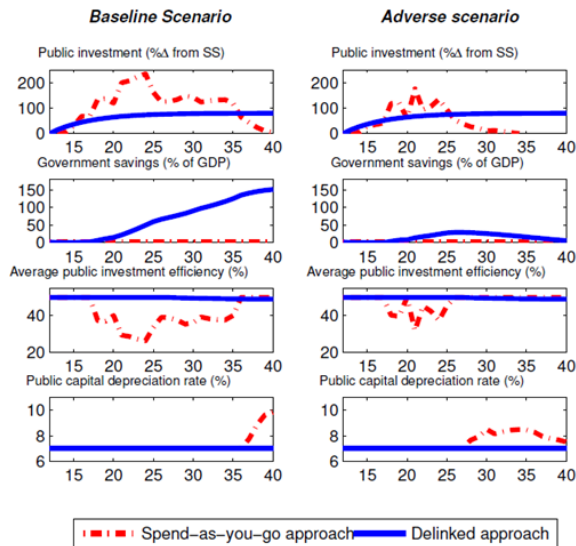
$$f_t^* = \max\left\{f_{\text{floor}}, f_{t-1}^* + \frac{f_{in,t}}{s_t} - \frac{f_{out,t}}{s_t}\right\},$$

where  $f^*$  is foreign financial asset,  $f_{in}$  and  $f_{out}$  represent fiscal inflows and outflows.

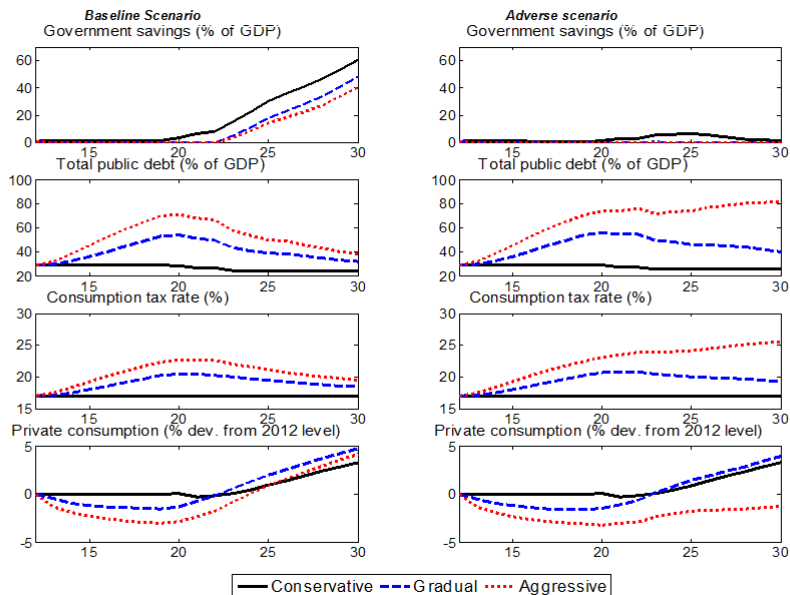
- The government has two public investment strategies: (i) spend-as-you-go approach; and (ii) a delinked/gradual scaling up approach. It can finance (part of) the scaling up through the sovereign wealth fund (in addition to the other fiscal and debt instruments)

$$gap_t = f_{out,t} - f_{in,t} + s_t (f_t^* - f_{t-1}^*).$$

# Given the volatility of the resource sector, smoothing public investment pays off...



# And the investment profile has important fiscal consequences...



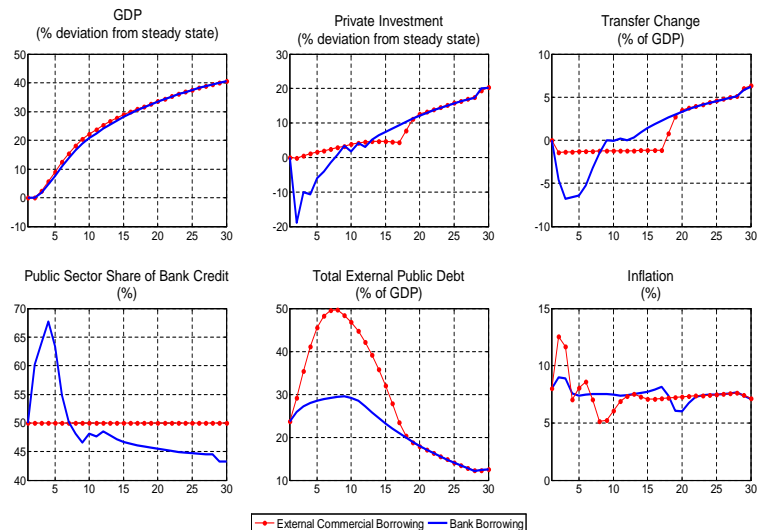
# Energy investment with a state-owned banking system in Ethiopia

- The government invests in infrastructure and energy: A big-push investment program in energy as envisioned by the GTP (Renaissance Dam).
- A state-owned energy sector exports electricity to neighboring countries and sells to firms and households at controlled, low prices (i.e., below shadow prices).
- A state-owned banking (SOB) sector that fixes interest rates and lends a large fraction of deposits to the public sector.
  - Some loans to the private sector at a controlled interest rate for investment in big projects. Loans may finance high- or low-return projects.
  - Large fraction of deposits lent to the government (Government effectively borrows at the real deposit rate).
  - Seigniorage and profits/losses of SOB enter the consolidated public sector budget constraint.
  - Private sector holds money and bank deposits, where the deposit rate is set by the government. The private capital account is closed.

# Energy investment with a state-owned banking system in Ethiopia

- Different initial returns: 20% for infrastructure investment vs. 30% for energy investment.
- 50% of bank credit is allocated to the public sector.
- The investment plan is not fully funded by concessional loans and grants, creating a financing gap:
  - More domestic bank borrowing would require a drastic fiscal adjustment of transfers, crowding out private investment in the short run, which is also the result of a sharp increase in inflation.
  - More external commercial borrowing would relax the national budget constraint avoiding difficult, large cuts in transfer payments and private investment crowding out. These positive effects, however, would be at the cost of significant increases in the ratios of non-concessional debt and total external debt to GDP.
- *Preferences*: separable in non-energy and energy consumption; *Technology*: Cobb-Douglas with energy capital as an input.

# Ethiopia: Domestic bank borrowing vs external borrowing



# Ethiopia: Domestic bank borrowing vs external borrowing

## Domestic bank borrowing

- Substantial fiscal adjustment is required to prevent macroeconomic instability. Transfers have to decrease by 7% of GDP within two years.
  - Bank deposits are approximately 20% of GDP, so the government can borrow at most another 9-10% of GDP by seizing all bank credit. This is not enough to finance the big-push investment program for more than a couple of years.
- Substantial crowding-out of private consumption and investment in the short run and sharp increase in inflation in the short run.
  - Given the closed private capital account, the lack of domestic financing available for firms translates into an increase in prices.

## External borrowing

- Macroeconomic stability does not require difficult, large cuts in transfer payments.
- No temporary crowding out of private investment and much smaller increase in inflation in the short run.



# Investing in electricity in Lesotho

- The government expects to invest in the construction of a hydroelectric plant. The energy supplied domestically is sold abroad.
- The stakes are high: the government needs to ensure that the investment is efficient and that demand from South Africa is sufficiently high.
- The state-owned energy sector produces electricity  $q_e$  linearly combining effective energy infrastructure  $\tilde{k}_e$  with technology  $a_e$

$$q_{e,t} = a_{e,0}\tilde{k}_{e,0} + a_e\theta \left( \tilde{k}_{e,t-1} - \tilde{k}_{e,0} \right),$$

where the first term represents the initial/existing technology and energy capital and the second is the new infrastructure project with capacity factor  $\theta$  and new technology.

## The energy sector

- The sector needs electricity purchased at a low price from abroad  $P_{e,t}^{buy}$  to produce additional electricity.

- Net sales of electricity

$$P_{e,t} q_{e,t}^{Domestic} + P_{e,t}^{sell} q_{e,t}^{Foreign} - P_{e,t}^{buy} (q_{e,t} - q_{e,0})(1 + loss\_factor)$$

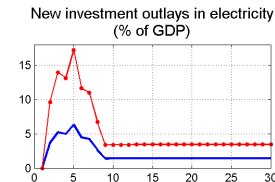
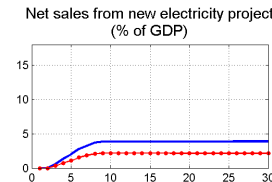
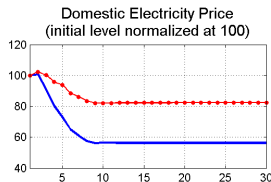
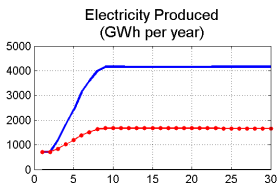
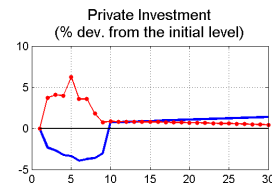
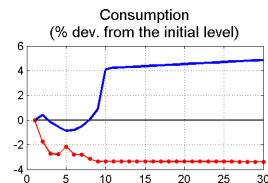
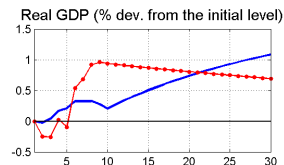
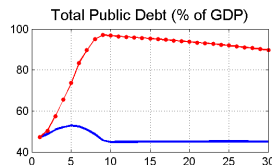
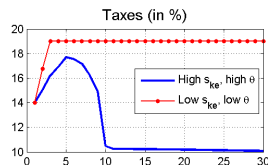
- Profit per unit of electricity sold abroad

$$P_{e,t}^* = P_{e,t}^{sales} - P_{e,t}^{buy} * (1 + loss\_factor) / (1 - \gamma)$$

- Lesotho's nominal GDP includes electricity output sold abroad (domestic electricity is consumed as an intermediate input)

$$y_t = P_{x,t} q_{x,t} + P_{n,t} q_{n,t} + P_{e,t}^* q_{e,t}^{Foreign}$$

# Investing with low efficiency and low capacity is costly...



## Improving revenue collection in Cambodia

- In addition to debt, the government can resort to alternative fiscal instruments: VAT on consumption, PIT on labor income, and CIT on firms' profits.
- However, a fraction of tax revenues are not collected or lost (e.g. tax expenditures, tax evasion).

$$\begin{aligned}
 \text{Gap}_t = & \underbrace{P_t \Delta b_t + \Delta d_{c,t}}_{\text{Borrowing}} + \underbrace{(\tau_t^c - \tau_o^c)(1 - \vartheta^c) P_t c_t}_{\text{VAT adjustment}} + \\
 & \underbrace{(\tau_t^l - \tau_o^l)(1 - \vartheta^l) w_t l_t}_{\text{PIT adjustment}} + \underbrace{(\tau_t^p - \tau_o^p)(1 - \vartheta^p) [\Pi_{n,t}^{op} + \Pi_{x,t}^{op}]}_{\text{CIT adjustment}}.
 \end{aligned}$$

- Improving tax collection allows Cambodia to rely less on debt financing.

# Improving revenue collection in Cambodia

## Real GDP (% $\Delta$ from SS) Cambodia

### *Baseline scenario*

Gradual 1.3-1.7 p.p.

Front-loaded 0.7-1.1 p.p.

### *Effective revenue collection scenario*

Gradual 1.3-1.8 p.p.

Front-loaded 0.8-1.2 p.p.

## Debt (% of GDP) Cambodia

### *Baseline scenario*

Gradual 36-52%

Front-loaded 36-54%

### *Effective revenue collection scenario*

Gradual 33%

Front-loaded 33%

*In addition to long term growth.*

## Investing in schools vs. roads

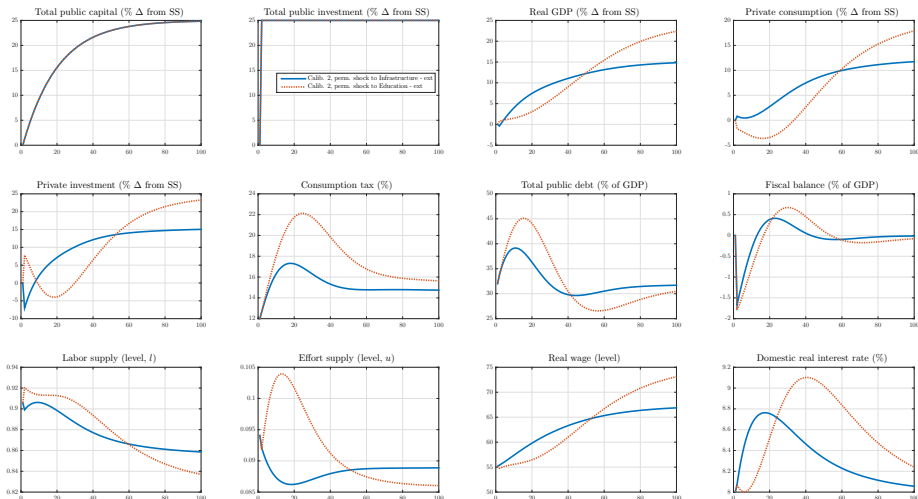
The government wants to permanently increase capital expenditures and its plan includes investment projects in social and economic infrastructure.

- Investing in education entails greater current expenditures (we assume O&M of schools are more expensive) and the benefits to the economy take time to materialize (a delay of 12y, mimicking the time spent in schools).
- On the other hand, the return on education capital is also greater than the return on standard economic infrastructure.
- Households can devote time to work and time to accumulate human capital.
- Human capital trapped in schools is the result of government-provided education infrastructure ( $z^e$ ), households' time devoted to education ( $u$ ), and the current level of human capital in the economy ( $e$ ).

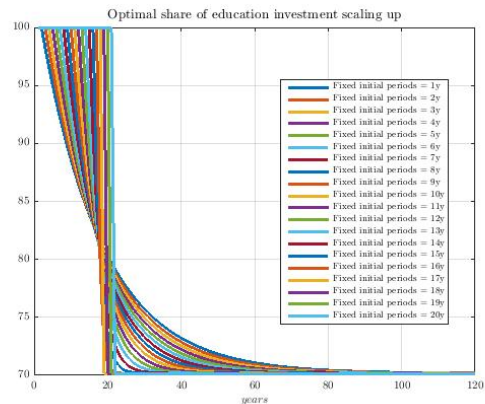
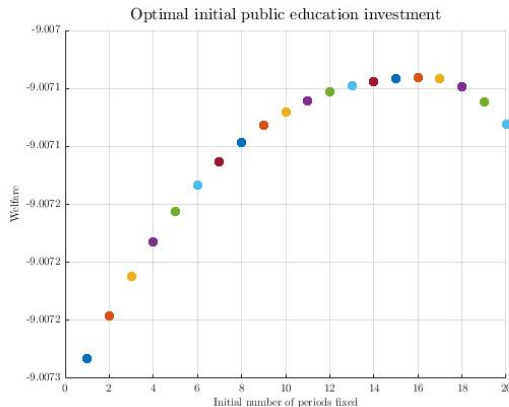
$$(1 + g) \xi_t = (1 - \omega) x i_{t-1} + a_e (z_{t-1}^e)^{v_e} (u_t e_t)^{(1-v_e)},$$

where a fraction  $\omega$  of the human capital moves from school to the labor force and  $v_e$  is the education elasticity to public education investment.

# Investing in schools takes time to materialize...



However, a benevolent planner interested in maximizing social welfare still wants to invest in education...

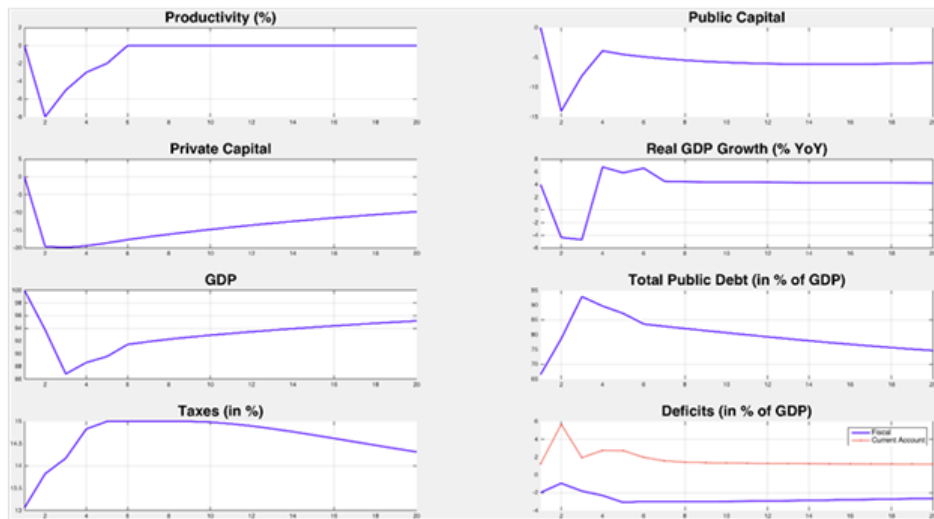




# The impact of a natural disaster

- The impacts of a natural disaster can be modeled in a DIG model as a loss of current output, a (permanent) loss of public and private capital, and a (temporary) decline of productivity.
  - The disaster strikes towards the end of the year and destroys 8% of current year output.
  - The disaster also wipes out 20% of the country's capital stock, which amounts to a loss of 56.8% of GDP (bringing total loss from the disaster to 64.8% of GDP).
  - The disruption in the functioning of the economic infrastructure is assumed to result in a persistent decrease in productivity, which is reduced in next three years by 5, 3, and 2% respectively.
  - Grants inflows offset part of the damages, which amount to 12% of GDP (5, 4, and 3% in first three years).
  - **Caveat:** The DIG model is built as a perfect foresight model and given the uncertainty around the frequency and severity of a natural disaster a framework with stochastic shocks would be favorable.

# Lower growth and higher debt for long...

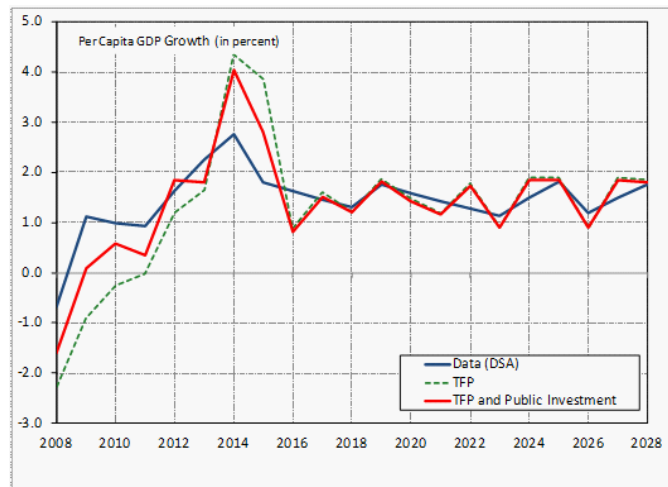


## Explaining growth projections in Togo

- How much of the growth projections is explained by public investment shocks instead of TFP shocks?
- *Invert* the non-linear model: calculate the model-implied structural shocks that will deliver the paths of some of the observed/projected macroeconomic variables.
  - Calibrate initial conditions to match Togo data in 2007-2008.
  - Assume shocks to TFP, public investment, terms of trade, grants, and remittances.... to explain the 2011 DSA paths for growth, public investment, exports, grants, and remittances....
  - Calculate the growth of GDP (pc) implied by simulating the model with (i) the implied TFP shocks only and (ii) the implied TFP and public investment shocks only

## Explaining growth projections in Togo

Although part of the growth projections is explained by public investment shocks, a large component is actually explained by TFP shocks



## Other extensions

- Incorporate uncertainty (shocks and parameters) more systematically while maintaining the *non-linear* structure, to construct confidence bands around debt trajectories.
- Enrich the model with other features (human capital, labor supply, habits persistence, maturity structure, etc.) and shocks (in trends, growth, preferences), to improve how to take the model to the data.
- Introduce other financing options such as public-private partnerships.

Can you **DIG** it?

*Asante sana!*